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Assessing impact of subjective demand beliefs on a dynamic duopoly electricity market game

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ABSTRACT

In the context of liberalized markets, market outcomes generally result from the strategic interactions of all market players. Generation company (Genco), as the distributed players, build their subjective demand evaluations (SDFs) about market for optimal bidding purpose. Due to the differences in terms of data availability and modeling techniques, subjective demand models held by various Gencos are heterogeneous and normally deviate from the real market model as well. The picture of a real electricity market game in Genco's eye is 'playing is believing'. Therefore, a question naturally comes to the table: how those SDFs with the heterogeneous manner impact individual player's decision and game results. To answer this question, this paper relaxes a conventional assumption, commonly used in the classical oligopolistic equilibrium model, that one correct and uniform demand knowledge is shared by all Gencos. The results suggest that the system equilibriums would be influenced by the Gencos' knowledge about market demand. The economic value of demand information is assessed regarding the system performances.

1. Introduction

In oligopoly Cournot electricity market games, market players, e.g. generation companies (Gencos) in this paper, simultaneously adjust their strategy (i.e. outputs) to increase their profits. The behaviors of Gencos are normally interpreted as a kind of equilibrium-oriented bidding evolution process. The notion of market equilibrium, e.g. Nash equilibrium, is commonly referred to a system state when dynamic interactions approach stable. Modeling market equilibrium is not only useful for system operator to monitor market power and assess market rules [1–3] but also could facilitate Gencos to identify their market power so as to bid in a reasonable way [1,3].

Many approaches have been proposed with the intention to model market players' behaviors and capture the resulting market equilibrium.

Taking its advantage of simple and flexible modeling, agent-based automatic learning, as one approach category, is used to simulate players' strategy decision making [4–8]. The reinforcement learning or its varieties are employed by all agents in the repeated market environment to weight predefined discrete actions via a long-term beneficial function. The system is said to converge if all players converge to the certain actions with which an optimal long-term beneficiary is assumed to be obtained.

Market equilibrium model, as another promising approach, also attracts much research interest. In equilibrium model, the rivals' behaviors are taken into account when on Genco develop its optimal strategies. When the interactions among Gencos approach stable it is said system reach the Nash equilibrium. Ref. [9] proposes a binary expansion scheme to find the Nash equilibrium in the short-term electricity markets. Ref. [10] proposes a compact formulation to find all pure Nash equilibriums in a pool-based electricity market with stochastic demands. Ref. [11] firstly employs the mathematical program with equilibrium constraints (MPEC) to model a single producer's behavior and then achieves the equilibrium by solving all MPEC simultaneously in a pool-based network-constrained electricity market.

However, all those approaches assume that there is one uniform and accurate market demand function available and shared by all Gencos. In the realistic electricity market, many stochastic factors, e.g. weather, demand side features, influence the real market demand functions. It is hard to have a commonly agreed function available for all Gencos. Each Genco has to construct its own market demand function, namely subjective demand function or belief

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in this paper, for bidding with all available data: history public data and its private information. Compared to real market model, in most cases Gencos more or less have evaluation errors or bias towards real market demand model, saying subjective demand error in this paper. Thus, it is important to study how the game will be changed with those demand errors, in term of, e.g. game equilibrium, steady state and system performance. The research questions in this paper are derived from the concerns of practical electricity market systems.

To our best knowledge, there has been relatively little prior work in this topic within electricity market system literature. Ref. [12] analyzes the impact of demand knowledge diversities on players' conjecture variation based bidding strategies and proposed a linear data filter method to alleviate system oscillations which are caused by such demand evaluation noises in a dynamic bidding process. Ref. [13] studies the impact of a mis-specified demand function on the steady state and the related attraction basins of a symmetric duopoly system. However, it did not consider the asymmetric system with heterogeneous players' behavior, which conform more to the reality.

In this paper, a duopoly game based on the subjective demand beliefs is set up. Our main result is that the system equilibriums are indeed influenced by the Gencos' knowledge about market. These influences are reflected by the two facts: first the SDFs could determine if the system equilibriums are local stable; and second if in the case of stable state the SDFs could change the positions of system equilibriums. Considering the possible knowledge deviations in the real electricity markets, the learning strategy by Gencos such as the adjustment length α in this paper should be cautiously chosen. By doing so, the unstable state of economic system and the resulting instabilities of the physical electricity network can be prevented.

The rest of this paper is organized as follows: Section 2 introduces the equilibrium model based on the subjective demand functions. Section 3 analyzes the stability conditions of the equilibriums in the proposed system. In Section 4, the system dynamics induced by demand errors are analyzed. The final section concludes the paper.

2. Genco's models with subjective demand functions

In this section, we introduce the game models which contain demand belief errors. For the models without belief errors, one can refer to the literature such as the work in [14–16]. Note that in this paper Gencos develop their strategic bidding responses based on a uniform price market clearing model.

2.1. Subjective demand beliefs

We assumes that there are a set of Gencos, denoted as I and a set of loads, denoted as J, dispersed among geographical locations. The node number is denoted as N. The demand evaluations by Genco i for the load j at the node n, denoted as f^e_{ijn}, is represented by the product of the reference demand of the load j located at the node n and an error coefficient ε_{ijn} $(0 < \varepsilon_{ijn} \leqslant 2)$. Then the subjective belief of the linear demand function by Genco i is described as follows:

$$f^e_{ijn} : p^e_{ijn} = \varepsilon_{ijn} \cdot (a^r_{jn} - b^r_{jn} \cdot D_{jn}) = \varepsilon_{ijn} \cdot (a^r_{jn} - b^r_{jn} \cdot (Q_{ijn} + Q_{\text{-}ijn})) \tag{1}$$

where a^r_{jn} and b^r_{jn} are the coefficients of the reference demand function; p^e_{ijn} is the evaluation price about the load j located at the node n by Genco i; D_{jn} is the demand consumption of the load j at the node n; Q_{ijn} and $Q_{\text{-}ijn}$ are, respectively, the supplies by Genco i and its competitors to the load j at the node n.

Since electricity market demands are dispersed among geographical locations, a vector to denote all subjective demand beliefs for Genco i at all nodes in system is defined as:

$$F^e_i = [f^e_{i11}, \cdots f^e_{ijn}, \cdots f^e_{IJN}]$$

where I, J and N are the total number of Gencos, loads and nodes.

In this paper, we assume that transmission capacities on all transmission lines are large enough so that the demand functions at different nodes can be integrated as one. Thus the node number is 1. Then the subscript of n and N in (1) can be omitted. Then for Genco i, its subjective demand function is transferred as:

$$p^e_i = \varepsilon_i \cdot (a^r - b^r \cdot (Q_i + Q_{-i})) \tag{2}$$

Fig. 1 shows an example about the relation between a reference demand function and the subjective demand functions held by different Gencos. In the figure, the value of the error coefficient, ε_i, represents the belief deviation of Genco towards the real demand function. Being smaller or larger than 1 indicates the cases of demand underestimation or overestimation respectively; the correct evaluation is the value 1. In this paper, we only discuss the cases that the error coefficients are static ones. The stochastic case is left for the future work.

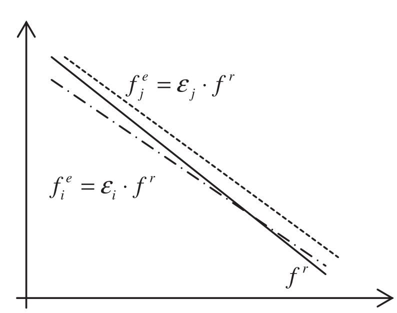

Fig. 1. An example of subjective demand beliefs.

2.2. Genco's behavior model with subjective demand beliefs

When Genco i outputs quantity Q_i, the production marginal cost takes the form as follows:

$$MC_i = \gamma_i \cdot Q_i + \beta_i$$

where $\gamma_i(\geqslant 0)$ and $\beta_i(\geqslant 0)$ are the coefficient of the cost curve for Genco i.

At time t, the profit maximization for a Genco is represented by the following optimization formula with the quantity $Q_{i,t}$ as the decision variable:

$$\begin{aligned} &\max\ P_{i,t} = p^e_{i,t} \cdot Q_{i,t} - \left(\tfrac{1}{2}\gamma_i \cdot Q^2_{i,t} + \beta_i \cdot Q_{i,t}\right) \\ &s.t.\ Q^{\min}_i \leqslant Q_{i,t} \leqslant Q^{\max}_i \end{aligned} \tag{3}$$

where $P_{i,t}$ is the expected profit of Genco i at time t. $p^e_{i,t}$ is the expected market price by Genco i at time t, which is an exogenous value for all Gencos. $Q^{\min}_i$ and $Q^{\max}_i$ are the upper and lower generation capacity limits for Genco i.

With the subjective demand functions defined in (2), the expected marginal profit of the Genco i for time $t+1$ is the derivative of profit calculated by (3) associated to the quantity $Q_{i,t+1}$:

$$\frac{\partial P_{i,t+1}(\varepsilon_i)}{\partial Q_{i,t+1}} = \varepsilon_i \cdot a^r - \beta_i - \gamma_i \cdot Q_{i,t+1} - 2 \cdot \varepsilon_i \cdot b^r \cdot Q_{i,t+1} - \varepsilon_i \cdot Q^e_{i,-i,t+1} \tag{4}$$

According to the first-order optimality condition, when letting the formula (4) equals to zero, the obtained quantity $Q_{i,t+1}$ is the expected optimal value with the demand belief error ε_i. It is represented as follows:

$$Q_{i,t+1} = (\varepsilon_i \cdot a^r - \beta_i - \varepsilon_i \cdot b^r \cdot Q^e_{i,-i,t+1})/(\gamma_i + 2 \cdot \varepsilon_i \cdot b^r) \tag{5}$$

The formula (5) indicates that the optimal quantity is dependent on both the behaviors of Genco's rivals $Q^e_{i,-i,t+1}$ and Genco's belief error about the market demand ε_i. That is to say the knowledge deviations about the demand would implicitly influence Genco's supply quantities at each bidding time.

Another behavior model is based on the assumption that player updates its strategy for the coming bidding time using local information which is based on the direction of the marginal profits at the previous bidding time with an adjustment step $\alpha(0 < \alpha \leqslant 1)$. It is also called the bounded rationality [17,18]. Such behaviors are represented as:

$$Q_{i,t+1} = Q_{i,t}\left(1 + \alpha \cdot \frac{\partial P_{i,t}(\varepsilon_i)}{\partial Q_{i,t}}\right) \tag{6}$$

The adjustment step α indicates that speed or step length by which player adjusts its strategy between two continuous bidding times. The smaller the value of adjustment step is, the higher possibility the dynamic system approaches to stable state; and vice versa. In the following sections, the efforts will be given to reveal the relation between the system equilibrium stability and the value of α when the Gencos' demand belief errors are taken into account.

Both in (4) and (6), the item $Q^e_{i,-i,t+1}$ should be evaluated. Regarding the evaluations about the rivals' behaviors for the coming bidding time, one simplest principle is that Genco i think that the rivals will not change their supply quantities, i.e. $Q^e_{i,-i,t+1} = Q_{-i,t}$. This expectation pattern is also called naïve expectation proposed by Cournot in [19].

Thus, the resulting heterogeneous duopoly game in which one Genco behave as the rule defined by (5) and the other follows the rule defined by (6) is formulated as follows:

$$Q_{1,t+1} = (\varepsilon_1 \cdot a^r - \beta_1 - \varepsilon_1 \cdot b^r \cdot Q_{2,t})/(2 \cdot \varepsilon_1 \cdot b^r + \gamma_1) \tag{7.1}$$

$$\begin{aligned} Q_{2,t+1} = Q_{2,t} \cdot (1 + \alpha \cdot (\varepsilon_2 \cdot a^r - \beta_2 - \varepsilon_2 \cdot b^r \cdot Q_{1,t} - 2 \cdot \varepsilon_2 \cdot b^r \\ \cdot Q_{2,t} - \gamma_2 \cdot Q_{2,t})) \end{aligned} \tag{7.2}$$

3. System equilibriums and their stabilities

The equilibriums of the given system (7) are obtained by solving the fixed points of the dynamical system when setting $Q_{i,t+1} = Q_{i,t}$. The system has two equilibriums indicated as follows:

$$O_1 = \left(\frac{-\beta_1 + \varepsilon_1\gamma_1}{2\varepsilon_1 b^r + \gamma_1}, 0\right)$$

and

$$O_o = \left(\frac{\varepsilon_1(a^r\gamma_2 + a^r b^r \varepsilon_2 + \beta_2 b^r) - \beta_1(2\varepsilon_2 b^r + \gamma_2)}{\gamma_1\gamma_2 + 2\gamma_1\varepsilon_2 b^r + 2\gamma_2\varepsilon_1 b^r + 3\varepsilon_1\varepsilon_2 b^{r^2}}, \frac{\varepsilon_2(a^r\gamma_1 + a^r b^r \varepsilon_1 + \beta_1 b^r) - \beta_2(2\varepsilon_1 b^r + \gamma_1)}{\gamma_1\gamma_2 + 2\gamma_1\varepsilon_2 b^r + 2\gamma_2\varepsilon_1 b^r + 3\varepsilon_1\varepsilon_2 b^{r^2}}\right) \tag{8}$$

The equilibrium point O_1 is not an effective equilibrium since it means that the Genco2 withdraw from the market at the end. The effective system equilibrium is the point O_o. The value of O_o shows that the system equilibrium is not determined by what strategies are adopted by Genco since the adjustment step α is not included in the equilibrium O_o, but determined by the Genco's beliefs about the demand. The following work in this paper will display how the system equilibrium is influenced by the knowledge error of the Genco.

The local stabilities of equilibriums O_1 and O_o are evaluated by judging the roots of the characteristic equations of the system's Jacobian matrix. The Jacobian matrix of the system (7) is developed as follows:

$$J(Q_1, Q_2) = \begin{bmatrix} 0 & -\frac{\varepsilon_1 b^r}{2\varepsilon_1 b^r + \gamma_1} \\ -\alpha Q_2 \varepsilon_2 b^r & 1 - \alpha Q_2(2\varepsilon_2 b^r + \gamma_2) - \alpha(Q_2(2\varepsilon_2 b^r + \gamma_2) - \varepsilon_2 a^r + \varepsilon_2 b^r Q_1) \end{bmatrix}$$

If and only if the magnitudes of eigenvalues of the above matrix at the certain equilibrium points are less than 1, the equilibriums are asymptotically stable.

It is proved that the Jacobian matrix associated with the equilibrium O_1 has two eigenvalues: $\lambda_1 = 1 + \alpha\left(\frac{\varepsilon_1 b^r \beta_2 + A\varepsilon_1\varepsilon_2 b^r + A\varepsilon_1\gamma_2}{2\varepsilon_2 b^r + c_2}\right)$ and $\lambda_2 = 0$. Since $|\lambda_1| \geqslant 1$, the point O_1 is not an stable equilibrium of the system (7).

Regarding to the stability of the equilibrium point O_o, the Jury stability criterion [20] is employed. If and only if the trace ($Tr(J)$) and determinant ($Det(J)$) of the Jacobian matrix $J(O_o)$ satisfy the relations as following:

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} Tr(J) \\ Det(J) \end{bmatrix} > -\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{9}$$

The region outlined by (9) describes the local stability region of the equilibrium O_O associated to the adjustment step α and the error coefficients $\varepsilon_i (i = 1, 2)$ when the other system parameters are given.

One numerical example is utilized in this section to show the impact of demand error coefficients on the stability region. In order to highlight the impact of demand error, the duopoly game is designed as such that two Gencos have the identical cost function data listed in the Table 1.

The reference demand function is shown in Table 2.

3.1. Perfect demand belief case

For the given system, if two Gencos have the perfect idea about the market demand, i.e. belief error coefficients of both Gencos are 1, the stability of the system equilibrium is just determined by the Genco's strategic behavior which is represented by the adjustment factor $\alpha \in (0, 0.0739]$. For this case, one knows one fact that the strategy adopted by Genco will impact if the dynamic system would enter the equilibrium state but not determine the system equilibrium value.

Table 1
Cost function of Gencos.

	$\gamma_{1,2}$ (€/MW^2 h)	$\beta_{1,2}$ (€/MW h)
Genco1/Genco2	0.025	3

Table 2
Reference demand.

	a^r (€/MW h)	b^r (€/MW^2 h)
Reference demand	35	0.018

3.2. Imperfect demand belief case

In the Fig. 2, the stability region of the system equilibrium is outlined in a three-dimensional space, also named error-strategy ($\varepsilon - \alpha$) space. The space underlying the surface is the new stability

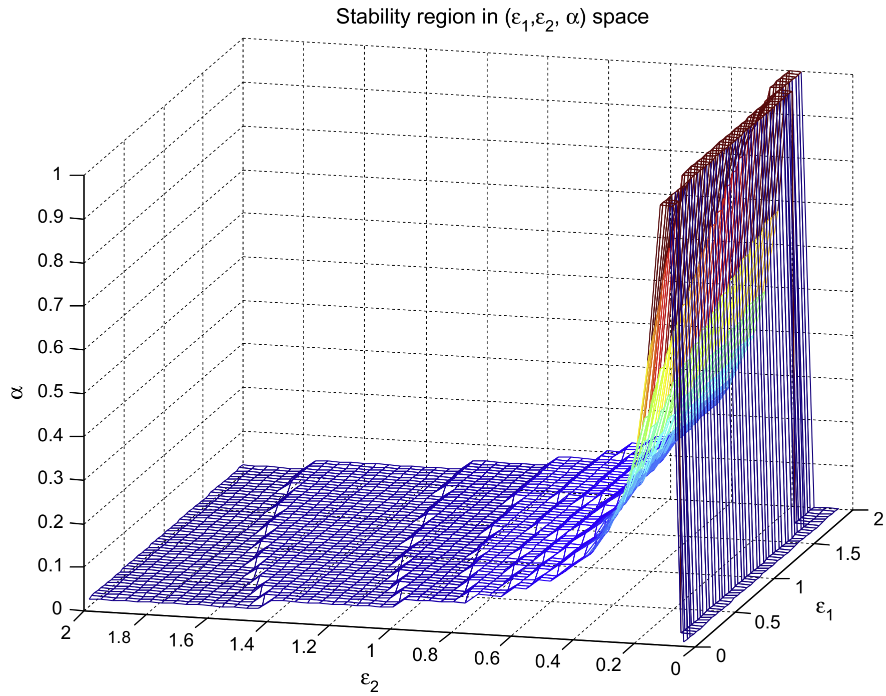

Fig. 2. The stability region defined in the error-strategy space.

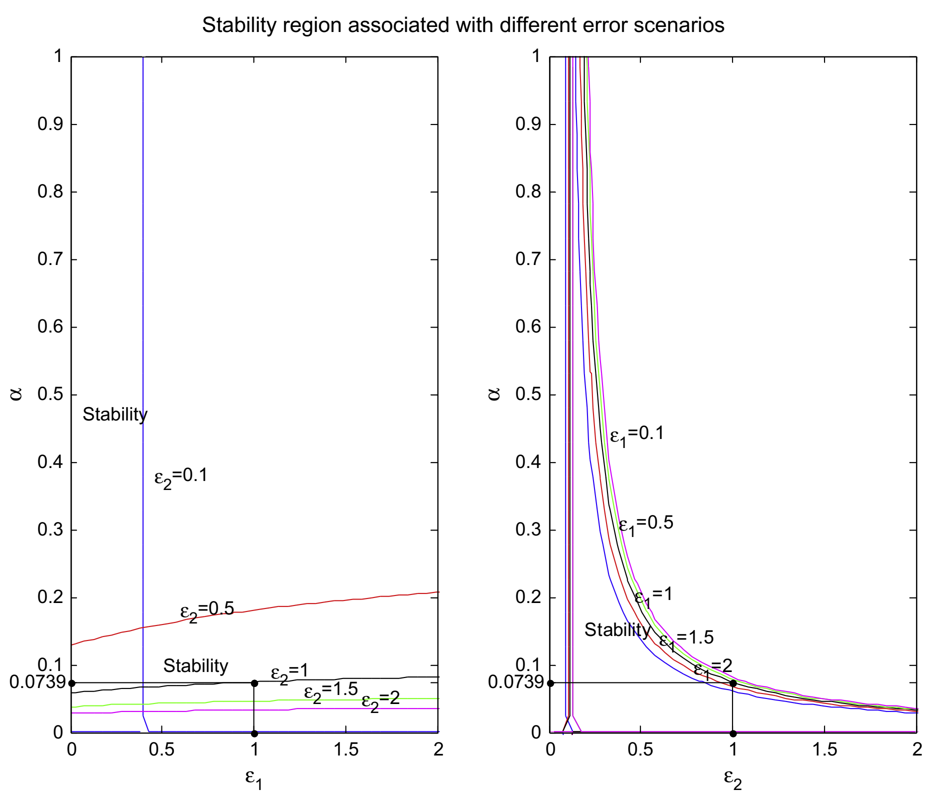

Fig. 3. The stability region defined in the sampled error-strategy space.

region. This region is different from the one in the case of perfect demand belief since the knowledge deviations of players re-define the stability region.

Fig. 3 displays the stability regions projected into the two two-dimensional spaces. The sampled curves with the different color represent the error coefficient of 0.1, 0.5, 1, 1.5 and 2 respectively. The stability regions in two sub-figures are labeled by 'stability'. The different color lines represent the situations from underestimates to overestimates by a Genco. Now we are going to analyze the impact of demand belief errors on the stability regions.

Firstly, the comparisons of two sub-figures show that the belief errors of Genco1 and Genco2 have the different impacts on the shape of stability defined by α. It is due to the facts that the two Gencos have the different behavior patterns. In the left sub-figure, the relation between α and ε_1 is changing with the different error levels of Genco2. The case when $\varepsilon_2 = 0.1$ appears special when comparing to the other four cases. In this case, when ε_1 is bounded by a certain value (approximately 0.5), α is available in the full range. That is to say, when both Gencos (extremely) underestimate the demand, the system is always stable no matter what the value of α is. This is because when ε_2 or ε_1 is very small the output of two Gencos are also very small (almost approaching zero in some cases), the system is always stable since the system is almost static (no bidding behaviors). However, this situation is not realistic since Gencos have no motivations to extremely underestimate the market demand at an unreasonable level. While, for the case that $\varepsilon_2 \geqslant 0.5$, the situations are different. With the error levels of Genco2 go higher, the stability ranges of α vs. ε_1 are shrunk. For instance, the area surrounded by the red line where $\varepsilon_2 = 0.5$ is bigger than the one surrounded by the pink line where $\varepsilon_2 = 2$. That is to say that the overestimation by Genco2 would shrink the stability range of α. While, for the case with a certain value of ε_2, the stability range of α would become larger and larger with the increasement of Genco1's error levels. So the impacts resulting from two Gencos' belief errors on the stability range of α show the opposite trends.

In the right sub-figure, firstly it is seen that stability regions in the different cases (outlined by the different color lines) keep the same shape but are placed in the coordinate axis with the slight differences. Concretely, the stability regions defined in the (ε_2, α) space are pushed to left moving with the increasement of Genco1's error levels ε_1. That is to say that the effective range of α is not much influenced by the value of ε_1 but mainly determined by the value of ε_2. For the case with the same value of ε_1, the smaller ε_2 match the larger range of α; and vice versa. Moreover, for the same value of α staying in the stability range, the larger the value of ε_1 is, the smaller the effective range of ε_2 is. These observations match the analyses about the left sub-figure.

So we can conclude that the stability range of α is more influenced by the Genco2's belief errors and less by the Genco1's errors. It's logical that since α represents the Genco2's behavior adjustment, Genco2's own knowledge about the market would definitely impact its behavior adjustment step length and consequently the stability region of system equilibrium.

The work in this paper reveal one fact that although the system equilibrium is not dependent on the value of α (the value of α would determine if the system can enter equilibrium state), but indeed influenced by the perception of Genco about the market. When ignoring such belief errors, the system probably displays the unexpected dynamic behavior. This observation is ignored by the previous literatures [20] due to the assumption of perfect information of player.

4. System dynamics induced by demand errors

In this section, we are going to study the system dynamics induced by Gencos' demand belief errors. The numerical cases shown in the Tables 1 and 2 are employed here.

4.1. System dynamics

In this section, we set the value of α as 0.065 which falls inside the stability region defined in (0, 0.0739] for the error-free system. With this value, in the Fig. 4, two sub-figures separately draw the bifurcation phenomena of Gencos' outputs evoked by one demand belief error while keeping another error coefficient as a fixed value, e.g. $\varepsilon_{i,j(i \neq j)} = 0.8, 1, 1.2$. Correspondingly, the results in the six cases are displayed in the two sub-figures. For instance, the case1 to case3 represent the bifurcation diagrams of two Gencos associated to ε_1 when $\varepsilon_2 = 0.8$, 1 and 1.2 respectively. The similar settings are defined for the case4 to case6 just by exchanging ε_1 and ε_2. For the purpose of showing the system performances clearly in the time scale, the dynamic bidding processes with the given α value and the different combination scenarios of the certain $\varepsilon_{i,j(i \neq j)}$ are, correspondingly, drawn in the Fig. 5.

Firstly, in the left sub-figure of the Fig. 4, the comparison of the three cases shows that from the underestimation to the overestimation by Genco2, the system outputs associated to the increasement of ε_1 change from stable to unstable. That is to say for the given α value when the Genco2 underestimates the demand the system equilibrium can always stay in the stable state. This is also

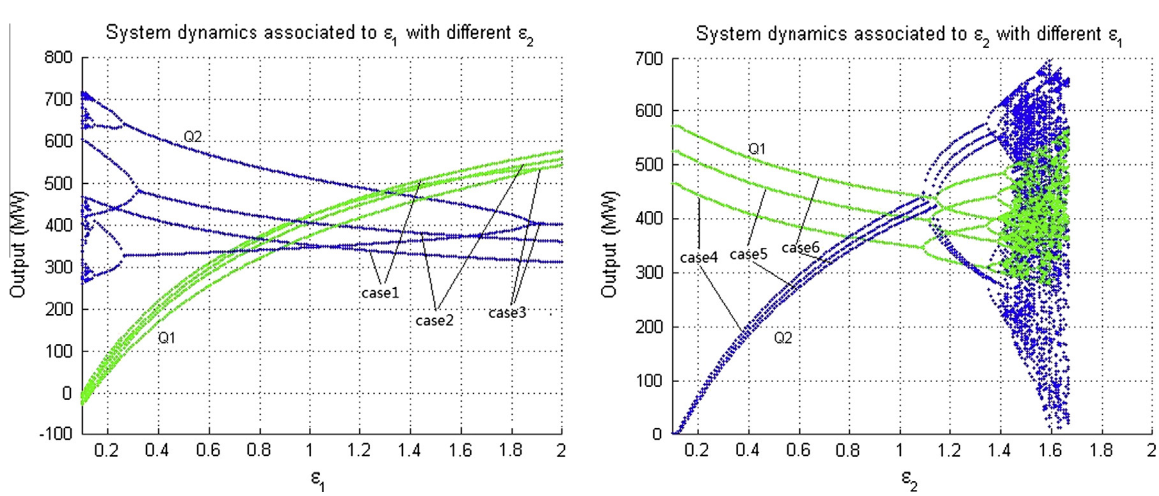

Fig. 4. System dynamics associated with different error scenarios.

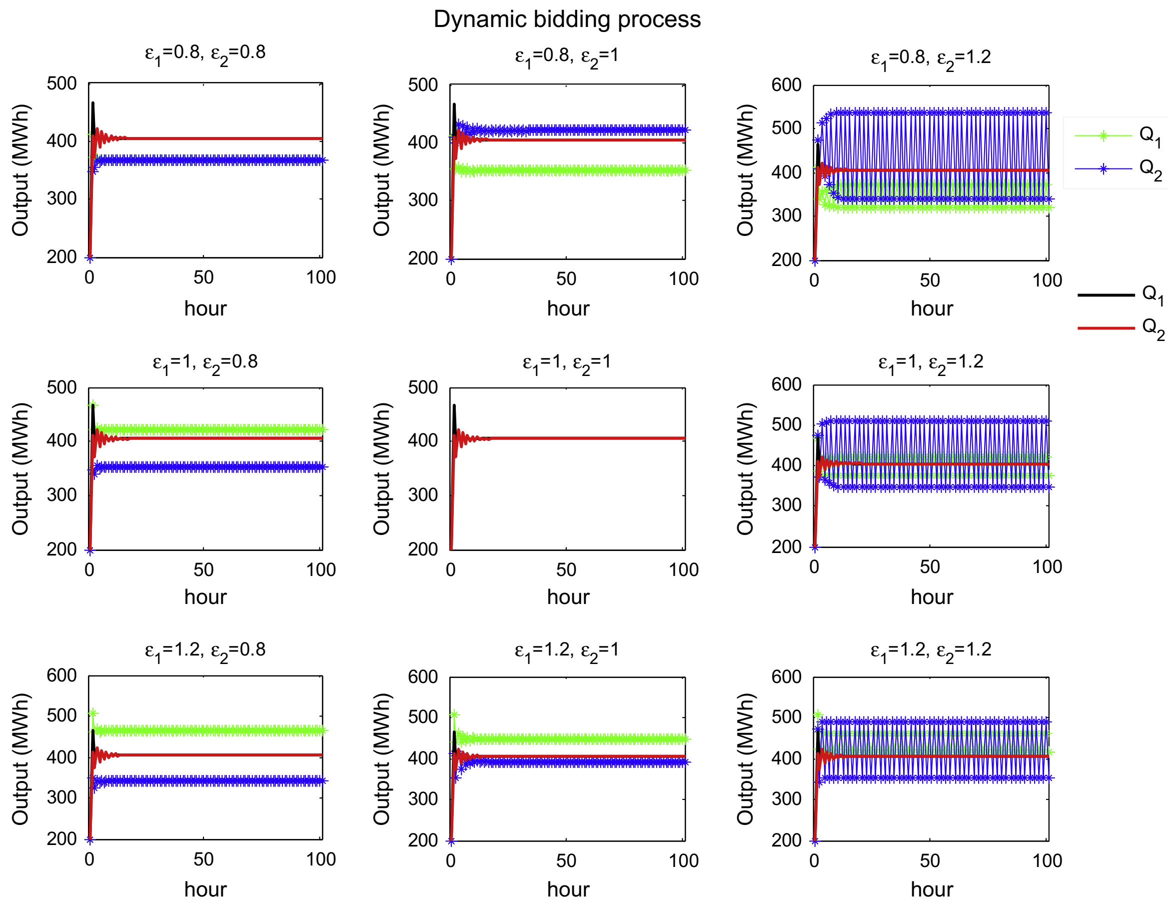

Fig. 5. Dynamic bidding processes in the different error scenarios.

can be seen from the three figures of the first column in the Fig. 5. The bidding processes finally move towards stabilization after a couple of oscillations. This phenomenon matches with the facts depicted in the left figure of Fig. 3 that when the value $\alpha = 0.065$ and when $\varepsilon_2 = 0.8$ the system is stable for every value of ε_1 within its effective range. When ε_2 increases to 1 and 1.2, the chaos take place for the system equilibrium regarding some ε_1 values in the various degree. For instance, when $\varepsilon_2 = 1$ the chaos or bifurcations appear with the very small ε_2 (almost smaller than 0.4); while the similar phenomenon of chaos occures regarding the larger range of ε_1 for the cases with $\varepsilon_2 = 1.2$. This can also be explained by the left sub-figure in the Fig. 3. It is seen that actually the overestimations by the Genco2 shrink the stability area at the coordination axis of α and ε_1.

In the right sub-figure of the Fig. 4, chaos occurs as well. In the three cases, the shapes and extent of the bifurcation are very similar. The larger values of ε_2 (larger than 1) result in the period-doubling bifurcations and a cascade of chaos for the subsequent periodic orbit. The three figures of the third column in the Fig. 5 display the dynamic bidding processes when $\varepsilon_2 = 1.2$. It can be seen that the system can't enter the stable state no matter what the value of ε_1 is since in the left sub-figure in the Fig. 3 the point constituted by $\varepsilon_2 = 1.2$ and $\alpha = 0.065$ falls outside of the stability range. Again, the comparisons of two sub-figures in the Fig. 4 reveal that the stability region is more sensitive to the demand belief error from the Genco2 (bounded rationality). These observations validate the analyses about the Fig. 3 from another perspective.

Moreover, both the gap value $|1 - \varepsilon_{i,j(i,j=1,2,i\neq j)}|$ and the distance value $|\varepsilon_i - \varepsilon_j|$ $(\forall i, j = 1, 2, i \neq j)$ would influence the stability of the system equilibrium in this numerical case. Logically, the smaller the gap $|1 - \varepsilon_{i,j}|$ is, the higher possibility the system equilibriums have to approach the stable. This is straightforwardly seen from the results in the Fig. 5. Likewise, the smaller the gap value $|\varepsilon_i - \varepsilon_j|$ is, the higher possibility the system equilibriums have to approach the stable. The logics behind are that the closer recognitions about the environment the players have, the higher possibility players have to reach the equilibrium.

Furthermore, seen from both the Figs. 4 and 5, the overestimations or underestimations of demand would push up or pull down the output of Gencos in the cases of stable equilibriums compared to the ones in the error-free situation. The system performances such as customer surplus and social welfare will be further analyzed in the following section.

4.2. System performances at subjective equilibrium

In the previous section, it is observed that the system can converge with some error levels, e.g. over- or underestimating demand by 5%. In such case the stable state of system equilibrium is a kind of subjective equilibrium (SE) whose existence and value are conditional on Gencos' demand knowledge perfection degree. In what follows the system performance in the SE will be studied. The equilibrium outputs and the associated profits for two Gencos, market price, and customer surplus are analyzed in the steady state compared to those in the error-free system and are listed in the Table 3 for the given $\alpha = 0.065$. We check the system results when under- or over-estimating is 5%, i.e. $\varepsilon_i^l = 0.95$ and $\varepsilon_i^h = 1.05$ for $i = 1, 2$.

The symbol % indicates the difference percentage between equilibrium profits in the studied cases in this paper to those in

Table 3
The system performances at subjective equilibrium.

		Error-free case	Single demand error case				Multiple demand errors case			
		$\varepsilon_1 = 1$ $\varepsilon_2 = 1$	$\varepsilon_1^l = 0.95$ $\varepsilon_2 = 1$	$\varepsilon_1^h = 1.05$ $\varepsilon_2 = 1$	$\varepsilon_1 = 1$ $\varepsilon_2^l = 0.95$	$\varepsilon_1 = 1$ $\varepsilon_2^h = 1.05$	$\varepsilon_1^l = 0.95$ $\varepsilon_2^l = 0.95$	$\varepsilon_1^l = 0.95$ $\varepsilon_2^h = 1.05$	$\varepsilon_1^h = 1.05$ $\varepsilon_2^l = 0.95$	$\varepsilon_1^h = 1.05$ $\varepsilon_2^h = 1.05$
Genco1 (best response)	q_1 (MW h)	405.1	392.9	416.5	408.6	401.7	396.5	389.6	420.2	413.1
	Profit (€/MW h)/%	6219.5	6153.4 −1.06%	6275.3 0.9%	6319.0 1.6%	6126.1 −1.5%	6252.8 0.54%	6060.0 −2.56%	6374.7 2.5%	6181.9 −0.6%
Genco2 (bounded rational)	q_2 (MW h)	405.1	408.6	401.7	392.9	416.5	396.5	420.2	389.6	413.1
	Profit (€/MW h)/%	6219.5	6319.0 1.6%	6126.1 −1.5%	6153.4 −1.06%	6275.3 0.9%	6252.8 0.54%	6374.7 2.5%	6060.0 −2.56%	6181.9 −0.6%
Market price (€/MW h)		20.41	20.57	20.27	20.57	20.27	20.7	20.42	20.42	20.13
Customer Surplus (€/MW h)		1158.0	1175.5	1141.6	1175.5	1141.6	1193.4	1158.7	1158.7	1125.4

the error-free case, i.e. $\% = \frac{P_i' - P_i^0}{P_i^0}\%, \forall i = 1, 2$, where P_i^0 and P_i' are the equilibrium profits of Genco i in error-free system and error system respectively.

The data in Table 2 reveals the following facts.

Firstly, in single-error case, at SE, under- or over-estimating demand by one Genco will accordingly lead its outputs lower or higher. Correspondingly, its profits are decreased or increased along with the opposite change of market price; meanwhile the customer surplus is increased or decreased. It is observed that another Genco with perfect demand evaluation accordingly raises or reduces its outputs and its profits are, thereby, raised or reduced. Especially in the case of underestimation, the correct Genco benefits from the higher market price induced by the demand underestimation. The lost profit of the error Gencos partially go to another Genco and partially go to the customer. In another word, the system surplus is redistributed among Gencos and load due to the imperfect demand evaluations by one player.

Secondly, at SE, the outputs and profits of one Genco with a certain error level are inversely impacted by anther Genco's demand evaluation errors. For example, in the cases that ε_1 is 0.95 and ε_2 is 0.95, 1 and 1.05 respectively, the outputs and profits of Genco1 display the decreasing trend.

Thirdly, in multiple-error case with the homogeneous demand error settings, i.e. $\varepsilon_1 = \varepsilon_2$, one interesting phenomenon is that when both Gencos simultaneously underestimating demand by 5%, their profits are increased by 0.54% although their outputs are decreased. The main reason is that the market price is raised by their capacity withholding behaviors. Consequently they benefit from such behaviors induced by their demand underestimations. If there is no information exchange occurring between two players, in this situation, the game is played in a tacit collusion manner. Thus the uncertainties about demand create the possibilities to collaboratively exercise the market power. It can also be assumed that in this situation Gencos maybe have no incentives to improve their demand evaluation precision to approach the real market function. Moreover Gencos' collusion behaviors indeed keep the system staying in a stable state, i.e. SE. Furthermore, in this situation customer surplus is also higher than the one in error-free system. By contrast, for the case in which both Gencos overestimate demand by 5%, their profits are lowered due to the fact that the market price decreases.

Finally, in multiple-error case with the heterogeneous demand error settings, i.e. $\varepsilon_1 \neq \varepsilon_2$, due to the symmetry of system and two Gencos' errors in spite of in two directions, the efforts to reduce or raise the market price by those two symmetric errors seems to be counteracted since the prices in those cases are very close to the one in the error-free system. By this reason the customer surplus almost does not change. Each Genco is impacted by its individual demand evaluation bias in terms of outputs and profits as expected. These observations indicate that the symmetrically opposite bias demand errors would result in the redistribution of profits between two Gencos but not with customer.

5. Conclusions

In this paper, the conventional model of a dynamic Cournot electricity market game where Gencos are assumed to hold a uniform and accurate belief towards market is replaced by a more realistic model with the subjective demand errors. As a consequence, the impact of such inaccurate demand belief from Gencos on a duopoly game is thoroughly studied. Our results show that the demand errors will reshape the stability regions of the original error-free system. Moreover, the stability regions in the proposed model react differently to Genco behaviors' variance. This feature is very important in designing Genco's adaptive strategy. Finally, a new equilibrium concept, namely subjective equilibrium, emerges. The system performances, regarding the equilibrium output, profit and customer surplus, at SE are analyzed.

In this paper, we restricted our analysis to deterministic models in which the inaccurate demand belief is assumed as fixed along game iterations. It gives a clear insight on the impact of the assumptions on the results of the dynamic Cournot game. The future work is to extend the analyses to stochastic models in which players exhibit time-varying belief errors.

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